

T. D. n° 1

Surfaces de réponse et plans pour mélanges

Exercice 1. Analyse canonique

We run a central composite design and fit a second-order model. The fitted coefficients are:

$$y = 86 + 9.2x_1 + 7.3x_2 - 7.8x_1^2 - 3.9x_2^2 - 6.0x_1x_2.$$

Perform the canonical analysis on this response surface.

Exercice 2. Détermination d'un maximum

Fit the second-order model to the fruit punch data of Example 5. Which mixture gives the highest appeal?

Exercice 3. Analyse canonique

The whiteness of acrylic fabrics after being washed at different detergent concentrations (.09 to .21 percent) and temperatures (29 to 41°) was measured and the following model was obtained ([PM84]):

$$y = -116.27 + 819.58x_1 + 1.77x_2 - 1145.34x_1^2 - .01x_2^2 - 3.48x_1x_2.$$

Perform the canonical analysis on this response surface.

Exercice 4. Rocket

Three components of a rocket propellant are the binder (x_1), the oxidizer (x_2), and the fuel (x_3). We want to find the mixtures that yield coefficients of elasticity (y) less than 3000. All components must be present and there are minimum proportions, so the investigators used a pseudocomponents design, with the following pseudocomponent values and results (data from [Kur66] via [Par78]):

x_1	x_2	x_3	y
1	0	0	2350
0	1	0	2450
0	0	1	2650
1/2	1/2	0	2400
1/2	0	1/2	2750
0	1/2	1/2	2950
1/3	1/3	1/3	3000
2/3	1/6	1/6	2690
1/6	2/3	1/6	2770
1/6	1/6	2/3	2980

1. Does this design correspond to any of our standard mixture designs?
2. Does it have an estimate of pure error?
3. Fit the second-order mixture model.
4. Is the estimated maximum above 3000?
5. Where is the estimated maximum, and where is the region that has elasticity less than 3000?

Exercice 5. Farines

Millers want to make bread flours that bake into large loaves. They need to mix flours from four varieties of wheat, so they run an experiment with different mixtures and measure the volume of the resulting loaves (ml/100g dough). The experiment was performed on 2 separate days, obtaining the following results (data from [DPL⁺93]):

Day 1					Day 2				
x_1	x_2	x_3	x_4	Volume	x_1	x_2	x_3	x_4	Volume
0	.25	0	.75	403	0	.75	0	.25	423
.25	0	.75	0	425	.25	0	.75	0	417
0	.75	0	.25	442	0	.25	0	.75	388
.75	0	.25	0	433	.75	0	.25	0	407
0	.75	.25	0	445	0	0	.25	.75	338
.25	0	0	.75	435	.25	.75	0	0	435
0	0	.75	.25	385	0	.25	.75	0	379
.75	.25	0	0	425	.75	0	0	.25	406
.25	.25	.25	.25	433	.25	.25	.25	.25	439

Analyze these data to determine which mixture of flours yields the largest loaves.

Exercice 6. Moteur

An experiment is performed to determine how a gasoline engine responds to various factors. The response of interest is CO emissions in grams per hour. The design factors are engine load, in Newton meters, range (30, 70); engine speed, in rpm, range (1000, 4000); spark advance, in degrees, range (10, 30); air-to-fuel ratio, dimensionless, range (13, 16.4); and exhaust gas recycle, in percent, range (0, 10). The experimental design has 46 observations in two blocks of 23 each. The design factors have been coded to the range $(-1, 1)$ in the table below (data from [DDPG94]).

Load	Speed	Advance	Ratio	Recycle	Block	Response
-1	-1	0	0	0	1	81
1	-1	0	0	0	1	148
-1	1	0	0	0	1	348

Load	Speed	Advance	Ratio	Recycle	Block	Response
1	1	0	0	0	1	530
0	0	-1	-1	0	1	1906
0	0	1	-1	0	1	1717
0	0	-1	1	0	1	91
0	0	1	1	0	1	42
0	-1	0	0	-1	1	86
0	1	0	0	-1	1	435
0	-1	0	0	1	1	93
0	1	0	0	1	1	474
-1	0	-1	0	0	1	224
1	0	-1	0	0	1	346
-1	0	1	0	0	1	147
1	0	1	0	0	1	287
0	0	0	-1	-1	1	1743
0	0	0	1	-1	1	46
0	0	0	-1	1	1	1767
0	0	0	1	1	1	73
0	0	0	0	0	1	195
0	0	0	0	0	1	233
0	0	0	0	0	1	236
0	-1	-1	0	0	2	100
0	1	-1	0	0	2	559
0	-1	1	0	0	2	118
0	1	1	0	0	2	406
-1	0	0	-1	0	2	1255
1	0	0	-1	0	2	2513
-1	0	0	1	0	2	53
1	0	0	1	0	2	54
0	0	-1	0	-1	2	270
0	0	1	0	-1	2	277
0	0	-1	0	1	2	303
0	0	1	0	1	2	213
-1	0	0	0	-1	2	171
1	0	0	0	-1	2	344
-1	0	0	0	1	2	180
1	0	0	0	1	2	280
0	-1	0	-1	0	2	548
0	1	0	-1	0	2	3046
0	-1	0	1	0	2	13
0	1	0	1	0	2	123
0	0	0	0	0	2	228
0	0	0	0	0	2	201
0	0	0	0	0	2	238

Analyze these data and describe how CO emissions depend on engine settings.

Exercice 7. Recommendations pratiques

Briefly describe an experimental design appropriate for each of the following situations.

- (a) Whole house air exchangers have become important as houses become more tightly sealed and the dangers of indoor air pollution become known. Exchangers are used primarily in winter, when they draw in fresh air from the outside and exhaust an equal volume of indoor air. In the process, heat from the exhausted indoor air is used to warm the incoming air. The design problem is to construct an exchanger that maximizes energy efficiency while maintaining air flow volume within tolerances. Energy efficiency is energy saved by heating the incoming air minus energy used to power the fan. There are two design variables: the pore size of the exchanger and the fan speed. In general, as the pore size decreases the energy saved through heat exchange increases, but for smaller pores the fan must be run faster to maintain air flow, thus using more energy. We have a current guess as to the best settings for maximum energy efficiency (pore size P and fan speed S). Any settings with 15% of P and S will provide acceptable air flow, and we feel that the optimum is probably within about 5% of these current settings.
- (b) Neuropeptide Y (NPY) is believed to be involved in the regulation of feeding and basal metabolism. When rat brains are perfused with NPY, the rats dramatically increase their food intake over the next 24 hours. Naloxone (NLX) may potentially block the effects of NPY. If so, it could be an important line of research in obesity studies. We wish to test the effect of four treatments, the factorial combinations of brain perfusion by either NPY or saline (as a control), and the subcutaneous injection of either NLX or saline (as a control) on 24-hour post-treatment food intake. We have available 32 male inbred, essentially similar rats.
- (c) We are trying to produce a new cleaning solvent for circuit boards. We anticipate that a combination of three standard solvents will work as well as the specialty solvent currently in use, but beyond knowing that we want each of the three to be at least 10% of the combination, we don't know how much of each to use.
- (d) Child development specialists are interested in factors affecting the ability of children to solve "ten questions" puzzles. In these puzzles the child is given a set of pictures, one of which has been chosen by the researcher. The child gets to ask questions that the researcher answers either yes or no; on the basis of these answers the child tries to determine which of the pictures has been chosen. The response the researchers are looking at is the number of

questions (ten maximum) that the child asks before determining the chosen picture. Two factors are under study: the number of pictures to choose from (either fifteen or twenty), and the familiarity of the objects in the pictures (either dinosaurs or birds, and oddly enough, I think the dinosaurs are the familiar objects!). The researchers have funds to study twelve children, and they expect substantial child to child variation. All children will do four puzzles, one of each type. They expect learning to take place, so that the later puzzles will generally be solved more quickly.

- (e) A fertilizer company is developing a rose fertilizer which consists of a nitrogen compound N, a phosphorus compound P, a potassium compound K, and an inert binder to hold it all together. (The binder can be disregarded in the experiment.) The company believes that there are optimum levels of N, P, and K to give best rose yield, and they believe that their current settings $N_0 = 6$, $P_0 = 6$, and $K_0 = 4$ (kg per 100 kg of fertilizer) are pretty close to optimal; probably each is within 10% of the optimal values. They want to find the optimal values.

Exercice 8. Lien le plus fort

Curing time and temperature affect the shear strength of an adhesive that bonds galvanized steel bars. The following experiment was repeated on 2 separate days. Twenty-four pieces of steel are obtained by random sampling from warehouse stock. These are grouped into twelve pairs; the twelve pairs are glued and then cured with one of nine curing treatments assigned at random. The treatments are the three by three factorial combinations of temperature (375°, 400°, and 450°F, coded $-1, 0, 2$) and time (30, 35, or 40 seconds, coded $-1, 0, 1$). Four pairs were assigned to the center point, and one pair to all other conditions. The response is shear strength (in psi, data from [Khu92]):

Temp.	Time	Day 1	Day 2
-1	-1	1226	1213
0	-1	1898	1961
2	-1	2142	2184
-1	0	1472	1606
0	0	2010	2450
0	0	1882	2355
0	0	1915	2420
0	0	2106	2240
2	0	2352	2298
-1	1	1491	2298
0	1	2078	2531
2	1	2531	2609

Determine the temperature and time settings that give strong bonds.

Exercice 9. Plans et ANOVA

For each of the following, briefly describe the design used and give a skeleton ANOVA.

- (a) National forests are managed for multiple uses, including wildlife habitat. Suppose that we are managing our multiple-use forest, and we want to know how snowmobiling and timber harvest method affect timber wolf reproductive success (as measured by number of pups surviving to 1 year of age over a 5-year interval). We may permit or ban snowmobiles; snowmobiles cover a lot of area when present, so we can only change the snowmobile factor over large areas. We have three timber harvest methods, and they are fairly easy to change over small areas. We have six large, widely dispersed forest sections that we may use for the experiment. We choose three sections at random and ban snowmobiles there. The other three sections allow snowmobiles. Each of these sections is divided into three zones, and we randomly assign one of the three harvest methods to each zone within each section. (Note that we do not harvest the entire zone; we merely use that harvest method when we do harvest within the zone.) We observe timber wolf success in each zone.
- (b) Some aircraft have in-flight deicing systems that are designed to prevent or remove ice buildup from the wings. A manufacturer wishes to compare three different deicing systems. This is done by installing the system on a test aircraft and flying the test aircraft behind a second plane that sprays a fine mist into the path of the test aircraft. The wings are photographed, and the ice buildup is estimated from interpretation of the photographs. They make five test flights for each of the three systems. The amount of buildup is influenced by temperature and humidity at flight altitude. The flights will be made at constant temperature (achieved by slightly varying the altitude); relative humidity cannot be controlled, but will be measured at the time of the flight.
- (c) We wish to study new varieties of corn for disease resistance. We start by taking four varieties (A, B, C, D) and cross them (pollen from type A, B, C or D fertilizing flowers from type A, B, C, or D), getting sixteen crosses. (This is called a diallel cross experiment, and yes, four of the sixteen “crosses” are actually pure varieties.) The sixteen crosses produce seed, and we now treat the crosses as varieties for our experiment. We have 48 plots available, 16 plots in each of St. Paul, Crookston, and Waseca. We randomly assign each of the crosses to one of the sixteen plots at each location.
- (d) A political scientist wishes to study how polling methods affect results. Two candidates (A and B) are seeking endorsement at their party convention. A random sample of 3600 voters has been taken and divided at random into nine sets of 400. All voters were asked if they support candidate A. However, before the question was asked, they were either told (a) that the poll is funded by candidate A, (b) that the poll is funded by candidate B, or (c) nothing. Due to logistical constraints, all voters in a given set (of 400) were given the

same information; the response for a set of 400 is the number supporting candidate A. The three versions of information were randomly assigned to the nine sets.

References

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